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NEWTONIAN COSMOLOGICAL MODEL IN NON- COMMUTATIVE CLASSICAL MECHANICS

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ABSTRACT

Non-commutative and non-commutative spaces using the concept of equations and laws of physics, is a useful tool to describe various physical problems. In this paper we describe Newton's Second Law in a non-commutative space presents. Then, using a variation principle of geodesic equation is obtained in general relativity and the Newtonian limit or the approximation of weak gravitational field and we studied for the equation of the following generalization of geodesic equation obtained in the weak gravitational field in a non-commutative space give approximation. However, we make a non-commutative classical mechanics interpretation. After describing a non-commutative classical mechanics, a Newtonian cosmological model of classical mechanics, is investigated.

Keyword: Newtonian Cosmological model, Non- Commutative Classical Mechanics

INTRODUCTION

Here, a Newtonian cosmological model of non-commutative classical mechanics is studied.As a non-commutative space in which diverse directions are to be found other ways to test cosmological model of particle motion in a way that non-commutative Newtonian cosmology principle is violated.It will be shown that the assumption of the non-commutative de Sitter $(\Lambda > 0)$ leads cosmological space for corrections to be applied on the Hubble constant. Obviously, if the parameter is non-commutative zero to all of the results in this paper, becomes the results of the Newtonian cosmological standard.

1 - de Sitter and anti de Sitter space at Newtonian limit

Einstein equation is as follows [15]:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu} \ .$$

(1)

In equation (1), $R_{\mu\nu}$ the Ricci tensor and R is the scalar curvature, and Λ is a universal constant of nature, which is called the cosmological constant. $T_{\mu\nu}$ is the energy-momentum tensor of the field material. In a vacuum and in the absence of Einstein's field equation in the presence of the material, $T_{\mu\nu}$ is zero cosmological constant statement is as follows:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 0 \ .$$

(2)

Small-scale distribution of matter in the universe is highly irregular. But looking at the size of the large-scale and larger world seems to be more and more uniform distribution of matter.

In fact, the available evidence suggests that very large scales the universe is isotropic carefully. Assuming that the universe has no center is preferable to the assumption of homogeneity in the structure of the macro world.The subjects were told to study the reasons of physical cosmology models are considered homogeneous and isotropic universe [22-24].According to the result of our discussions, it will lead to the cosmological principle according to which all parts of the world at a specific time and space seems to be the same for all directions in equal space anywhere. De Sitter and anti de Sitter space solutions of Einstein equation (2) are for a homogeneous and isotropic universe. The overall shape of the space - time is as follows: [4, 25]:

$$ds^2 = c^2\left(1 - \frac{\Lambda r^2}{3}\right)dt^2 - \frac{dr^2}{\left(1 - \frac{\Lambda r^2}{3}\right)} - r^2\left(d\theta^2 + \mathrm{Sin}^2\theta d\Phi^2\right) .$$

(3)

In the case of negative metric (3) an anti-de Sitter space specifies the metric for positive values (3) de Sitter space is called. It is gone in Newtonian limit $c \to \infty$ and $\Lambda \to 0$, but $c\Lambda$, the product remains finite in the limit value. With respect to the metric (3) and also $g_{\circ\circ}$ will be as follows :(4)

$$g_{\circ\circ} = 1 - \frac{\Lambda r^2}{3} \quad .$$

Newtonian potential $\phi(x)$ is determined as follows:

$$\phi = -\frac{\Lambda c^2 r^2}{6} \quad .(5)$$

It will be a Hamiltonian system:

$$H = \frac{1}{2m} p^i p^i - \frac{m\Lambda c^2 r^2}{6} \quad .(6)$$

Equation (6) can be also achieved with a different way described in reference 25. Equation Hamilton - Jacobi for a particle of mass m in general relativity is as follows [25]:

$$g^{\mu\nu} \frac{\partial S}{\partial x^\mu} \frac{\partial S}{\partial x^\nu} = m^2 c^2 \ .(7)$$

In equation (7), is called the S Hamilton - Jacobi.

Hamilton - Jacobi equation depends on the metric (3) for a particle with mass of m and angular momentum will be:

$$\frac{1}{c^2}\left(1 - \frac{\Lambda r^2}{3}\right)^{-1}\left(\frac{\partial S}{\partial t}\right)^2 - \left(1 - \frac{\Lambda r^2}{3}\right)\left(\frac{\partial S}{\partial r}\right)^2 = m^2 c^2 . (8)$$

Now we assume that the answer of equation (8) is as follows:

$$S(t,r) = -Et + S_r(r) \quad ,(9)$$

Where E is a constant value. By placing (9) (8) will be obtained:

$$\left(1-\frac{\Lambda r^2}{3}\right)\left(\frac{dS_r(r)}{dr}\right)^2 = \frac{E^2}{c^2\left(1-\frac{\Lambda r^2}{3}\right)} - m^2c^2 \, . \quad (10)$$

E fixed by writing as follows:

$$E = mc^2 + H \quad ,(11)$$

The purpose of energy H is the non-relativistic particle, equation (10) will be:

$$\left(1-\frac{\Lambda r^2}{3}\right)^2\left(\frac{dS_r(r)}{dr}\right)^2 = \frac{\left(mc^2+H\right)^2}{c^2} - m^2c^2\left(1-\frac{\Lambda r^2}{3}\right) \, . \quad (12)$$

Slightly simplified equation (12) can be written:

$$\left(1-\frac{\Lambda r^2}{3}\right)^2\left(\frac{dS_r(r)}{dr}\right)^2 = m^2c^2\frac{\Lambda r^2}{3} + 2mH + \frac{H^2}{c^2} \, . \quad (13)$$

For low speed is $H << mc^2$, equation (13) becomes as follows:

$$\left(1-\frac{\Lambda r^2}{3}\right)^2\left(\frac{dS_r(r)}{dr}\right)^2 = m^2c^2\frac{\Lambda r^2}{3} + 2mH \, . \quad (14)$$

On the other hand, the weak gravitational field approximation is $\Lambda r^2 << 1$, and equation (14) will be:

$$\left(\frac{dS_r(r)}{dr}\right)^2 = m^2c^2\frac{\Lambda r^2}{3} + 2mH \, .(15)$$

p defined as

$$p = \frac{dS_r(r)}{dr} \quad ,(16)$$

Equation (15) can be written as:

$$H = \frac{p^2}{2m} - \frac{m\Lambda c^2 r^2}{6} \quad .(17)$$

In reference 23, equations (6) and (17) have been obtained with an argument based on Newtonian mechanics.

The equations of motion of the device described by Hamiltonian (17) are obtained, we have:

$$m\frac{d^2\vec{r}}{dt^2} = m\frac{\Lambda c^2}{3}\vec{r} \quad , \quad (18)$$

That $\vec{r} = (x, y, z)$.

Equation (18) is similar to Newton's Second Law for a particle of mass m, which has been affected by an external force of $\vec{F} = m\frac{\Lambda c^2}{3}\vec{r}$. It is for the anti-de Sitter space $\Lambda < 0$ and equation (18) is as follows:

$$m\frac{d^2\vec{r}}{dt^2} = -m\frac{|\Lambda| c^2}{3}\vec{r} \quad . \quad (19)$$

The $\vec{F} = -m\frac{|\Lambda|c^2}{3}\vec{r}$ force is a force-absorbing particles and will fluctuate around the source:

$$\vec{r} = \vec{r}_{\circ}\,\mathrm{Sin}\left(\sqrt{\frac{|\Lambda|c^2}{3}}t\right) \quad ,(20)$$

$\vec{r}_{\circ}$ is a fixed vector and independent of time.

In this case $(\Lambda < 0)$ will be the distance from the source

$$r_{\Lambda<\circ}(t) = r_{\circ}\left|\mathrm{Sin}\left(\sqrt{\frac{|\Lambda|c^2}{3}}t\right)\right| \quad ,(21)$$

The de Sitter space $\Lambda > 0$ and equation (18) will have the response as follows:

$$\vec{r} = \vec{r}_{\circ}\,e^{\sqrt{\frac{\Lambda c^2}{3}}t} \quad ,(22)$$

That $\vec{r}_{\circ}$ is a fixed vector and independent of time. In this case $(\Lambda > 0)$, the distance from the source will be:

$$r_{\Lambda>\circ}(t) = r_{\circ}\,e^{\sqrt{\frac{\Lambda c^2}{3}}t} \quad , (23)$$

The Hubble constant is defined as

$$H = \frac{\dot{r}}{r} \quad , (24)$$

The de Sitter universe, the Hubble constant will be equal to:

$$H = \sqrt{\frac{\Lambda c^2}{3}} \quad .(25)$$

According to the definition of the Hubble constant to the de Sitter equation (23) can be written as:

$$r_{\Lambda>\circ}(t) = r_{\circ}e^{Ht} \quad .(26)$$

2 Formulation of Newtonian cosmology in a non-commutative phase space

According to above mentioned, the formulation described by the Hamiltonian (17) in a non-commutative phase space. Then we have:

$$\begin{aligned} m\ddot{x} &= \frac{m\Lambda c^2}{3}x + m^2\theta^{1j}\frac{\partial^2\phi}{\partial x^j\partial x^k}\dot{x}^k \\ &= \frac{m\Lambda c^2}{3}x + m^2\left(\theta^{12}\frac{\partial^2\phi}{\partial y\partial x^k}\dot{x}^k + \theta^{13}\frac{\partial^2\phi}{\partial z\partial x^k}\dot{x}^k\right) \\ &= \frac{m\Lambda c^2}{3}x - \frac{m^2\Lambda c^2}{3}\left(\theta^{12}\dot{y} + \theta^{13}\dot{z}\right) \end{aligned}$$

$$= \frac{m\Lambda c^2}{3}x - \frac{m^2\Lambda c^2}{3}\left(\theta^3\dot{y} - \theta^2\dot{z}\right) \quad ,(27)$$

$$m\ddot{y}=\frac{m\Lambda c^2}{3}y+m^2\theta^{2j}\frac{\partial^2\phi}{\partial x^j\partial x^k}\dot{x}^k$$
$$=\frac{m\Lambda c^2}{3}y+m^2\left(\theta^{21}\frac{\partial^2\phi}{\partial x\partial x^k}\dot{x}^k+\theta^{23}\frac{\partial^2\phi}{\partial z\partial x^k}\dot{x}^k\right)$$
$$=\frac{m\Lambda c^2}{3}y-\frac{m^2\Lambda c^2}{3}\left(\theta^{21}\dot{x}+\theta^{23}\dot{z}\right)$$

(28)
$$=\frac{m\Lambda c^2}{3}y-\frac{m^2\Lambda c^2}{3}\left(-\theta^3\dot{x}+\theta^1\dot{z}\right)\quad ,$$

$$m\ddot{z}=\frac{m\Lambda c^2}{3}z+m^2\theta^{3j}\frac{\partial^2\phi}{\partial x^j\partial x^k}\dot{x}^k$$
$$=\frac{m\Lambda c^2}{3}z+m^2\left(\theta^{31}\frac{\partial^2\phi}{\partial x\partial x^k}\dot{x}^k+\theta^{32}\frac{\partial^2\phi}{\partial y\partial x^k}\dot{x}^k\right)$$

(29)
$$=\frac{m\Lambda c^2}{3}z-\frac{m^2\Lambda c^2}{3}\left(\theta^2\dot{x}-\theta^1\dot{y}\right)\quad .$$

Since solving the system of equations (27) to (29), it is difficult to assume that the non-commutative parameter only along the z- vector is the zero, and $\vec{\theta}$ vector can be as follows:

(30)
$$\vec{\theta}=\left(\theta^1=\theta^{23}=0\,,\,\theta^2=\theta^{31}=0\,,\,\theta^3=\theta^{12}=\theta\right),$$

In this case the system of equations (27) to (29) in the form of easier are as follows:

(31)
$$m\ddot{x}=\frac{m\Lambda c^2}{3}x-\frac{m^2\Lambda c^2\theta}{3}\dot{y}\quad ,$$

(32)
$$m\ddot{y}=\frac{m\Lambda c^2}{3}y+\frac{m^2\Lambda c^2\theta}{3}\dot{x}\quad ,$$

(33)
$$m\ddot{z}=\frac{m\Lambda c^2}{3}z\quad .$$

In case $\theta=0$, the system of equations (31) to (33) becomes the equation (18) that the responses were studied.

In this case it is not possible for the system of equations (31) to (33) in response to have a form as:

(34)

$$\vec{r} = \vec{r}_{\circ} R(t) \quad ,$$

Which is consistent with the cosmological principle. In other words, there is a non-commutative parameter, θ, in the homogeneous and isotropic break down the $x - y$ plane.

Now we are looking for answers to equations (31) to (33). Since the non-commutative parameter, θ, affects only the screen $x - y$ itself in order to facilitate the response, it is assumed that along the z vector, we have an answer as:

(35)

$$z(t) = 0 \quad ,$$

By defining α and β parameters in the form of

(36)

$$\alpha = \frac{\Lambda c^2}{3} \quad ,$$

(37)

$$\beta = m\alpha\theta \quad ,$$

Equations (31) and (32) are in the following simple form:

(38)

$$\ddot{x} = \alpha x - \beta \dot{y} \quad ,$$

(39)

$$\ddot{y} = \alpha y + \beta \dot{x} \quad .$$

Considering the responses of equations (38) and (39) as

(40)

$$x = x_{\circ} e^{\omega t} \quad ,$$

(41)

$$y = y_{\circ} e^{\omega t} \quad ,$$

And placing them in equations (38) and (39) we have:

$$\begin{cases} (\omega^2 - \alpha) x_{\circ} + \beta\omega y_{\circ} = 0 \quad , \\ -\beta\omega x_{\circ} + (\omega^2 - \alpha) y_{\circ} = 0 \quad . \end{cases}$$

(42)

The condition of non-obvious solution to a homogeneous system of equations (42) to limit the amount of frequency, ω, leads to:

(43)

$$\omega=\sqrt{\alpha}\left[\left(1-\frac{\beta^2}{2\alpha}\right)\pm\sqrt{\left(1-\frac{\beta^2}{2\alpha}\right)^2-1}\right]^{\frac{1}{2}} .$$

Given the small size of non-commutative parameter, θ, if the computing power second only is kept in equation (43) will be:

(44)

$$\omega=\sqrt{\alpha}\left(1-\frac{\beta^2}{4\alpha}\mp\frac{\beta}{2\alpha}\sqrt{-\alpha}\right) .$$

For the anti-de Sitter, $(\Lambda<0)$, included in equation (44), and the imaginary, according to equations (40) and (41) as well as will have the fluctuations of shift mode. On the other hand, for the de Sitter, $(\Lambda>0)$, ω in equation (44) will be:

$$\omega=\omega_R\pm i\omega_I$$

(45)

$$=\sqrt{\alpha}\left(1-\frac{\beta^2}{4\alpha}\right)\pm i\frac{\beta}{2} .$$

In this case, our responses are as follows:

$$\begin{cases} x=e^{\omega_R t}\,\mathrm{Cos}\,\omega_I t \\ y=-e^{\omega_R t}(1-\frac{\beta^2}{8\alpha})\,\mathrm{Sin}\,\omega \end{cases} \quad (46)$$

and

$$\begin{cases} x=e^{\omega_R t}\,\mathrm{Sin}\,\omega_I t \\ y=e^{\omega_R t}(1-\frac{\beta^2}{8\alpha})\,\mathrm{Cos}\,\omega_I t \end{cases} .$$

(47)

The path we have spiral directions and the cosmological principle is violated. According to equation (35) and equation (46), $r(t)$ will be:

(48)

$$r(t)=r_\circ e^{\omega_R t}\sqrt{1-\frac{\beta^2}{4\alpha}\mathrm{Sin}^2\omega_I t} .$$

Another $r(t)$ value according to the equations (35) and (47) obtained as follows:

(49)

$$r(t)=r_\circ e^{\omega_R t}\sqrt{1-\frac{\beta^2}{4\alpha}\mathrm{Cos}^2\omega_I t} .$$

The Hubble constant is corrected in the equation (25) as follows:

(50)

$$H_{nc} = \omega_R = \sqrt{\alpha}\left(1 - \frac{\beta^2}{4\alpha}\right) \quad .$$

It should be noted that the higher the Hubble constant corrections sentences appear with the kinetics (Time dependence**)**.

3 strong θ limit

Strong θ limit in equations (31) to (33) can cause using limit process $\frac{m\Lambda c^2}{3} \to 0$ and $\theta \to \infty$ as long as $\left(\frac{m\Lambda c^2}{3}\right) m\theta$ remains finite sure, we have:

(51)

$$m\ddot{x} = -\frac{\Lambda c^2}{3} m^2 \theta \dot{y} \quad ,$$

(52)

$$m\ddot{y} = \frac{\Lambda c^2}{3} m^2 \theta \dot{x} \quad ,$$

(53)

$$m\ddot{z} = 0 \quad .$$

Equations (51) to (53) are similar to the equations of motion of a charged particle In the presence of a uniform magnetic field along the z axis are on the $x - y$ plate. In fact, equations (51) to (53) Λ, so play the role of charge and θ of magnetic fields. If the particle velocity is zero along the z axis the path of the particle in the $x - y$ plane will be as a circle. The sign of Λ specifies the direction of circular particle clockwise or counterclockwise on the $x - y$ plane.

CONCLUSION

Generalization of physics laws and equations is of the growing importance in non-commutative spaces. If Newton's Second Law extended for a central force field and classical equations obtained for a three-dimensional isotropic harmonic oscillators to the first problem to be solved in terms of non-commutative parameter, in the case of non-commutative parameter is zero, all results for the three-dimensional isotropic harmonic oscillators will become an oscillator results of classical dynamics Isotropic three-dimensional space. Geodesic equation is formulated in a weak gravitational field approximation in a place with interesting properties.

The equation of time is lacking inversion symmetry. To avoid time reversal symmetry breaking is essential that non-commutative parameter θ, turn under the transformation t → - t at the same time as θ → -θ. Newtonian motion of the particle paths are such that test cosmological model, where the violation is the principle of cosmology. It is concluded that the assumption of the non-commutative cosmological space where De Sitter leads to apply corrections on the Hubble constant. Obviously, if the parameter is not considered as non-commutative zero, all results becomes the results of Newtonian cosmology standard.

REFERENCES:

1. Juan M. Romero, J. A. Santiago and J. David Vergara, Phys. Lett .A **310**, 9 (2003).
2. A. Boulahoual and M. B. Sedra , J. Math. Phys **44**, 5888 (2003).
3. M. I. Beciu, arXiv:gr–qc/0305077v1 (2003).
4. Juan M. Romero and J. A. Santiago ,Mod. Phys. Lett. A **20** , 781(2005).
5. Tom W. B. Kibble and Frank H. Berkshire, "Classical Mechanics ", 5th Edition , Imperial College Press , London(2004).
6. Stephen T. Thornton and Jerry B. Marion , " Classical Dynamics of Particles and Systems", 5th Edition, Thomson. Brooks/Cole, (2004).
7. H. S. Snyder , Phys . Rev. **71** , 38 (1947).
8. C. N. Yang , Phys. Rev. **72**, 874 (1947).
9. R. J. Szabo , Quantum Field Theory on Noncommutative Spaces, Physics Reports, **378**, 207(2003).
10. N. Seiberg and E. Witten, String Theory and Noncommutative Geometry , JHEP 9909 (1999) 032, hep-th /9908142.
11. Z. F. Ezawa , Quantum Hall Effects : Field Theorical Approach and Related Topics (World Scientific, 2000).
12. Jerzy Plebanski and Andrzej Krasinski, "An Introduction to General Relativity and Cosmology", Cambridge University Press, (2006).
13. Oyvind Gron and Sigbjorn Hervik , "Einstein's General Theory of Relativity ", Springer, (2007).
14. N. M. J. Woodhouse , "General Relativity" ,Springer, (2007).
15. Moshe Carmeli, "Classical Fields : General Relativity and Gauge Theory", John Wiley & Sons, (1982).
16. P. A. M. Dirac, " General Theory of Relativity", John Wiley & Sons , (1975).
17. Bernard F. Schutz, " A First Course In General Relativity", Cambridge University Press, (1985).
18. Norbert Straumann , "General Relativity and Relativistic Astrophysics", Springer –Verlag(1984).
19. Robert M. Wald, "General Relativity", The University of Chicago Press, (1984).
20. Arif Akhundov and Anwar Shiekh, EJTP **5**, No.17 (2008) 1-16.
21. M. M. Sheikh – Jabbari , Phys. Rev. Lett . **84** (2000) 5265.
22. Hans C. Ohanian, "Gravitation and Spacetime", W. W. Norton & Company. Inc.(1976).
23. Matts Roos, "Introduction to Cosmology", Third Edition, John Wiley, (2003).
24. Andrew R. Liddle and, David H. Lyth "Cosmological Inflation and Large – Scale Structure ", Cambridge University Press, (2000).
25. G. W. Gibbons and C. E. Patricot, Class. Quantum. Grav. **20** , 5225(2003).